

# ORBITAL PARAMETERS FOR THE OGO-E GEO-CORONAL HYDROGEN EXPERIMENT

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BY

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NASA**GODDARD SPACE FLIGHT CENTER**  
**GREENBELT, MARYLAND**

ORBITAL PARAMETERS  
FOR THE OGO-E  
GEO-CORONAL HYDROGEN EXPERIMENT

by

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July 1965

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ABSTRACT

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This document presents and applies methods for obtaining certain necessary orbital and spacecraft parameters for meaningful data analysis of the geo-coronal hydrogen measurement experiment (Experiment No. 22) on the fifth in Orbiting Geophysical Observatory Series, namely the OGO-E.

Techniques are presented for determining as a function of time: 1) orbit inclination with respect to the ecliptic plane, 2) the angle between the orbit's angular momentum vector and the sun's position vector, 3) the half angular dimension of the earth viewed from the satellite, 4) the solar array angle and, 5) the angle between the projection of the sun on the x-y plane of the experiment's coordinate system and the x-axis of that system.

The calculations were made using a numerical integration general purpose interplanetary trajectory program (ITEM). Consequently, the changes in the orbit due to perturbations are reflected in the calculations of the quantities mentioned above.

The results of computer runs are presented in this document.

*Hecht*

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## NOMENCLATURE

$a$	Semi-major axis.
$a_e$	Radius of the earth.
$C_E$	The angle between the OPEP $x_E$ -axis and the projection of the sun vector on the OPEP $x_E$ - $y_E$ plane. (See Figure 2.)
$e$	Eccentricity.
$i$	Inclination.
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors along the $x, y, z$ axes of the equatorial inertial coordinate system.
$\bar{i}', \bar{j}', \bar{k}'$	Unit vectors along the $x', y', z'$ axes of the inertial ecliptic coordinate system.
$\bar{i}_b, \bar{j}_b, \bar{k}_b$	Unit vectors along the $x_b, y_b, z_b$ axes of the satellite fixed coordinate system.
$\bar{i}_E, \bar{j}_E, \bar{k}_E$	Unit vectors along the $x_E, y_E, z_E$ axes of the OPEP coordinate system.
$\bar{i}_p, \bar{j}_p, \bar{k}_p$	Unit vectors along the $x_p, y_p, z_p$ axes of the solar array coordinate system.
$M$	Mean anomaly.
$\bar{N}$	Unit angular momentum vector.
$\bar{r}$	Position vector of the satellite.
$S'$	The angle between normal to the orbit and the position vector of the sun relative to the earth.
$\bar{S}_{Ep}$	Sun vector projection on the OPEP $x_E$ - $y_E$ plane.
$\bar{U}_{SE}$	Unit vector from earth to sun in the $x, y, z$ coordinate system.
$x, y, z$	Axes of the inertial equatorial coordinate system.
$x', y', z'$	Axes of the inertial ecliptic coordinate system.

$x_b, y_b, z_b$	Axes of the satellite fixed coordinate system.
$x_E, y_E, z_E$	Axes of the OPEP coordinate system.
$x_p, y_p, z_p$	Axes of the solar array system.

### Greek Symbols

$\delta$	Half angular dimension of the earth viewed from the satellite.
$\epsilon$	The mean obliquity = $23^{\circ}44'37''$ for 1966 (Ref. 2 ).
$i$	The orbit inclination with respect to the ecliptic plane.
$\phi_p$	The solar array angle (Fig. 2).
$\psi_E$	The OPEP angle, the angle between the $x_E$ and $x_b$ axes.
$\omega$	Argument of perigee.
$\Omega$	Right ascension of the ascending node.

### Matrices

A	A transformation matrix defined by equation 8.
B	A transformation matrix defined by equation 9.
C	= B A, see equation 10.
X	A vector with components in the inertial equatorial coordinate system.
$X_b$	A vector with components in the satellite fixed coordinate system.
$X_E$	A vector with components in the OPEP coordinate system.
$X_{ES}$	Position vector of the sun in the OPEP system.
$X_S$	Position vector of the sun in the inertial equatorial coordinate system.

ORBITAL PARAMETERS  
FOR THE OGO-E  
GEO-CORONAL HYDROGEN EXPERIMENT

INTRODUCTION

The Orbiting Geophysical Observatory (OGO) satellites form a family of satellites each designed to carry up to fifty experiments. The OGO-E is one in this series. The discussion here concerns an experiment (No. 22) on the OGO-E which will make optical measurements of the thickness of the geo-coronal hydrogen. It is theorized that hydrogen is escaping from the earth. The purpose of the experiment is to determine the gradient of the hydrogen population around the earth and also to investigate the interaction caused by extra-terrestrial effects.

This document gives methods for determining the following quantities useful in post flight data analysis:

1. The angle  $\xi$  between the ecliptic plane and the orbit plane (Fig. 1).
2. The angle  $S'$  between the normal to the orbit plane and a vector from the earth to the sun (Fig. 1).



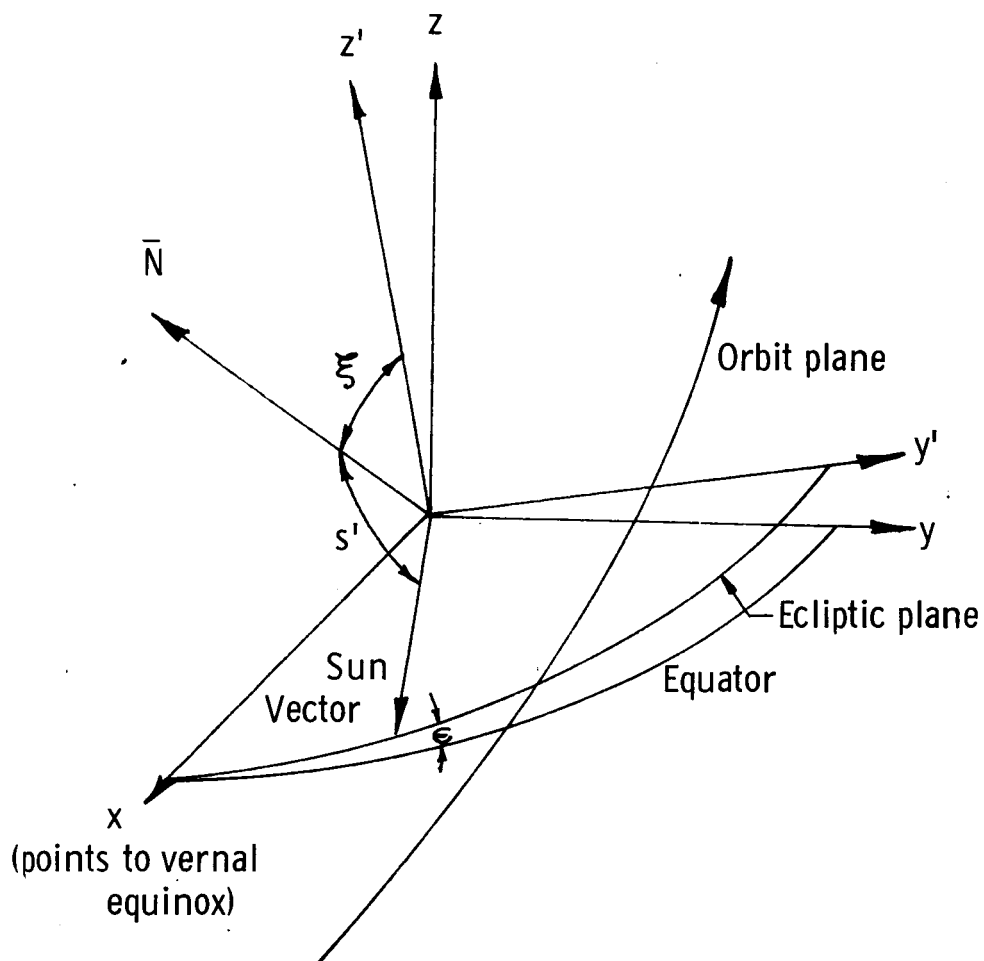


Figure 1.  
Angles  $\xi$  and  $s'$

3. The half angular dimension  $\delta$  of the earth as viewed from the satellite as a function of time.
4. The solar array angle  $\varphi_p$  between the normal to the sun-lit side of solar array and the  $y_b$ -axis of the satellite fixed coordinate system (Fig. 2).

5. The angle  $C_E$  between the projection of the sun vector on the  $x_E$ - $y_E$  plane of the orbit plane experiment package (OPEP) coordinate system and the  $x_E$ -axis of that coordinate system. The OPEP coordinate system,  $x_E$ ,  $y_E$ ,  $z_E$  is shown on Figure 2.

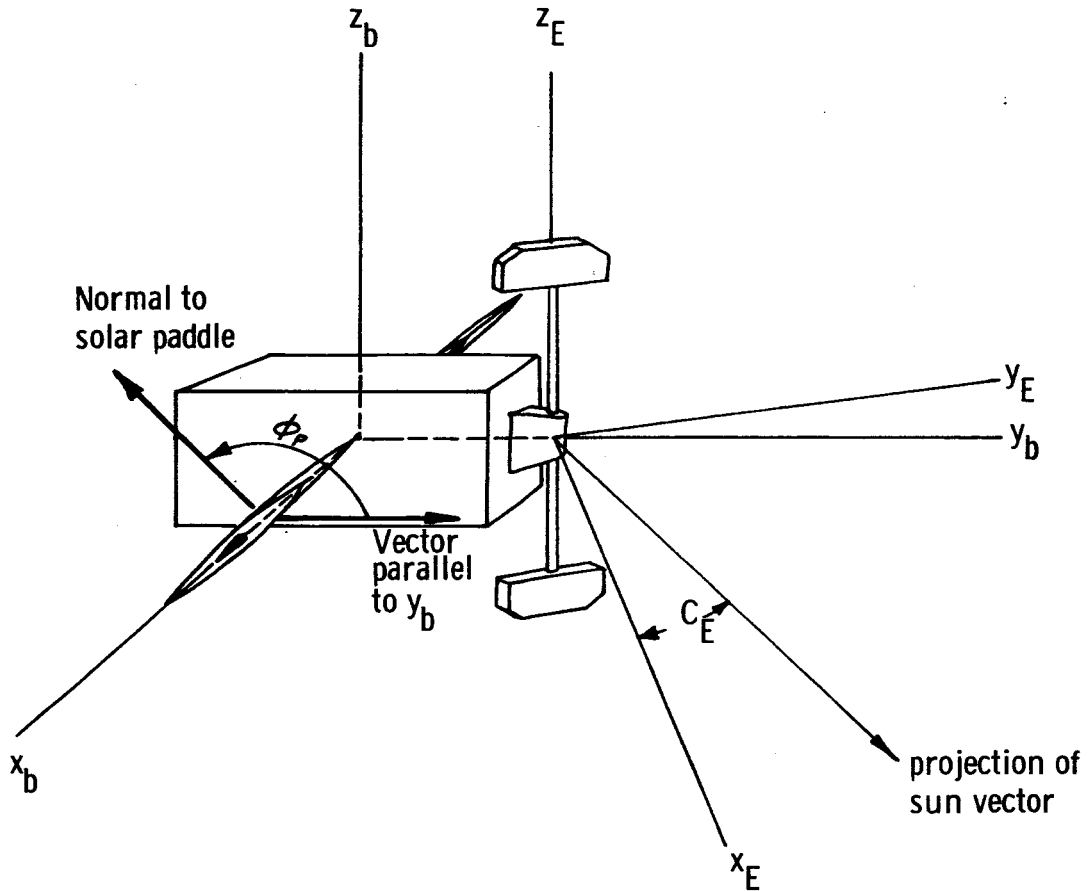


Figure 2.  
Angles  $\varphi_p$  and  $C_E$

## I. ASSUMPTIONS

The OGO-E will be placed into a highly eccentric orbit with an apogee height approximately 148,000.0 km. The injection conditions for OGO-E are assumed to be the same as the nominal conditions for OGO-A which was launched September 4, 1964 (Ref. 1).

---

Table I.

OGO-E Injection Conditions

Geocentric latitude	=	20°744573 S
Longitude	=	111°11923 E
Height	=	279.2517 km
Speed	=	10.716286 km per sec
Azimuth	=	66°445986 from north
Flight path angle	=	1°452111

---

The osculating classical orbital elements associated with these injection conditions are shown in Table II.

---

Table II

Orbital Elements

Semi-major axis,  $a = 12.514687$  earth radii

Eccentricity,  $e = .91666199$

Inclination,  $i = 30^{\circ}910496$

Argument of perigee,  $\omega = 313^{\circ}657536$

Right ascension of the ascending node,  $\Omega = 195^{\circ}74031$

Mean anomaly,  $M = 8^{\circ}7160632$

Epoch = August 15, 1966 at  $5^h 30^m$  U.T.

---

## II. MATHEMATICAL MODEL

### A. Orbit Plane Geometry

The angles  $\xi$  and  $S'$  are related to the orbit in space and the sun. These angles are considered as orbit related because the position of the spacecraft in the orbit does not enter into the calculations.

#### 1. The Inclination with respect to the Ecliptic Plane $\xi$ .

The first problem is to determine the angle between the normal to the ecliptic plane and the normal to the orbit plane, i.e. the inclination of the orbit with respect to the ecliptic plane. The angle can simply be written as:

$$\xi = \cos^{-1} (\bar{N} \cdot \bar{k}'). \quad (1)$$

where  $\bar{N}$  = Unit angular momentum vector which is normal to the orbit.

$\bar{k}'$  = Unit vector normal to the ecliptic plane.

$$\bar{k}' = -\bar{j} \sin e + \bar{k} \cos e$$

$e = 23^{\circ}44'37''$ , mean ecliptic angle for 1966 (Ref. 2).

$\bar{j}, \bar{k}$  = unit vectors along the y and z axes of the equatorial inertial coordinate system.

## 2. Angular Momentum - Sun Angle, S'.

The angle S' is defined by:

$$S' = \cos^{-1} (\bar{N} \cdot \bar{U}_{SE}) \quad (2)$$

where  $\bar{U}_{SE}$  = The unit position vector of the sun measured from earth. The coordinates of the sun are taken from an ephemeris.

The use of  $\bar{N}$  in equations (1) and (2) takes into account the motion of the orbit due to perturbations when used in a numerical integration scheme where for each integration step  $\bar{N}$  is computed. Consequently, the motion  $\bar{N}$  reflects both the secular and periodic perturbations. For a single orbit there is little change in  $\bar{N}$ ; hence, it can be stated that the angles  $\bar{S}$  and S' are dependent on the motion of the plane of the orbit. The angle S' is also dependent on the position of the sun.

### B. Spacecraft Angles

#### 1. Half Angular Dimension of the Earth, $\delta$ .

The angle,  $\delta$  is easily determined from the following expression:

$$\delta = \sin^{-1} \frac{a_e}{|r|} \quad 0^\circ \leq \delta \leq 90^\circ \quad (3)$$

where  $a_e$  = 6378.165 km radius of the earth (Ref. 3).

## 2. The Solar Array Angle, $\varphi_p$ .

The solar array angle,  $\varphi_p$ , is based solely on the position of the solar array with respect to the main structure on the satellite. The angle  $\varphi_p$  is measured counter-clockwise from the  $y_b$ -axis to the  $y_p$ -axis, as observed from the positive  $x_b$ -axis (Fig. 2). The planned range for the variation of  $\varphi_p$  for OGO-E is  $\frac{\pi}{2} \leq \varphi_p \leq \frac{3\pi}{2}$ , (Refs. 4, 5). Therefore,  $\varphi_p$  may be found from the formula

$$\varphi_p = \cos^{-1} (\bar{j}_p \cdot \bar{k}_b) + \frac{\pi}{2} \quad (4)$$

where  $\bar{j}_p$  = Unit vector normal to solar array surface (sunlit side).

$$\bar{k}_b = -\frac{\bar{r}}{|\bar{r}|}, \quad \bar{r} \text{ is the satellite position vector.}$$

## 3. OPEP $x_E$ -Axis - Sun Projection Angle, $C_E$ .

The angle  $C_E$  is somewhat more complicated in that two coordinate transformations are required involving three coordinate systems. The three coordinate systems in question are:

a).  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ , inertial equatorial system,

where  $x$ ,  $y$ ,  $z$  form the right-handed coordinate system in which  $x$  points to the vernal point,  $z$  points to the north pole and  $y$  completes the set.

$$b) \quad X_b = \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}, \text{ the satellite fixed coordinate system,}$$

where  $x_b$ ,  $y_b$ ,  $z_b$  form a right-handed coordinate system with the origin at the center of gravity of the satellite in which  $z_b$  points to the center of the earth from the satellite,  $x_b$  lies parallel to the solar array axis and  $y_b$  is orthogonal to  $x_b$  and  $z_b$  and points towards the orbit plane experiment package (OPEP) end of the satellite (Fig. 2).

$$c). \quad X_E = \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix}, \text{ the OPEP coordinate system,}$$

where  $x_E$ ,  $y_E$ ,  $z_E$  form a right-handed coordinate system in which  $z_E$  is parallel to  $z_b$  and the  $x_E - y_E$  plane is parallel to the  $x_b - y_b$  plane and has the angle  $\psi_E$  between the  $x_b$  and  $x_E$  axes as shown in the following figure.



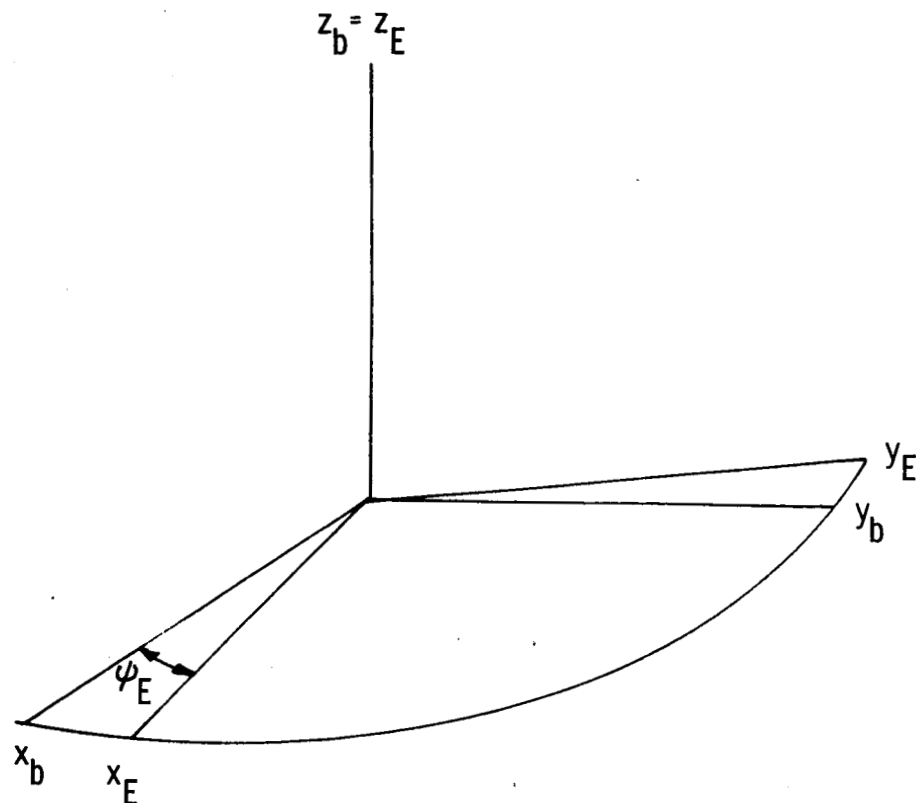


Figure 3.  
Angle  $\phi_E$

The three coordinate systems are related as follows:

$$X_b = AX \quad (5)$$

$$X_E = BX_b \quad (6)$$

$$\text{Then } X_E = CX \quad (7)$$

where A, B, C, are transformation matrices and

$$C = BA$$

In the three coordinate systems described above, let:

$\bar{i}, \bar{j}, \bar{k}$  = unit vectors along  $x, y, z$ .

$\bar{i}_b, \bar{j}_b, \bar{k}_b$  = unit vectors along  $x_b, y_b, z_b$ .

$\bar{i}_E, \bar{j}_E, \bar{k}_E$  = unit vectors along  $x_E, y_E, z_E$ .

Now, the matrix A is defined by

$$A = \begin{bmatrix} \bar{i}_b \cdot \bar{i} & \bar{i}_b \cdot \bar{j} & \bar{i}_b \cdot \bar{k} \\ \bar{j}_b \cdot \bar{i} & \bar{j}_b \cdot \bar{j} & \bar{j}_b \cdot \bar{k} \\ \bar{k}_b \cdot \bar{i} & \bar{k}_b \cdot \bar{j} & \bar{k}_b \cdot \bar{k} \end{bmatrix}. \quad (8)$$

The matrix B is:

$$B = \begin{bmatrix} \cos \psi_E & \sin \psi_E & 0 \\ -\sin \psi_E & \cos \psi_E & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

Therefore,

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad (10)$$

$$\text{where } c_{11} = (\bar{i}_b \cdot \bar{i}) \cos \psi_E + (\bar{j}_b \cdot \bar{i}) \sin \psi_E$$

$$c_{12} = (\bar{i}_b \cdot \bar{j}) \cos \psi_E + (\bar{j}_b \cdot \bar{j}) \sin \psi_E$$

$$c_{13} = (\bar{i}_b \cdot \bar{k}) \cos \psi_E + (\bar{j}_b \cdot \bar{k}) \sin \psi_E$$

$$c_{21} = -(\bar{i}_b \cdot \bar{i}) \sin \psi_E + (\bar{j}_b \cdot \bar{i}) \cos \psi_E$$

$$c_{22} = -(\bar{i}_b \cdot \bar{j}) \sin \psi_E + (\bar{j}_b \cdot \bar{j}) \cos \psi_E$$

$$c_{23} = -(\bar{i}_b \cdot \bar{k}) \sin \psi_E + (\bar{j}_b \cdot \bar{k}) \cos \psi_E$$

$$c_{31} = (\bar{k}_b \cdot \bar{i})$$

$$c_{32} = (\bar{k}_b \cdot \bar{j})$$

$$c_{33} = (\bar{k}_b \cdot \bar{k})$$

Since the position of the sun must be defined in the OPEP coordinate system, equation (7) is used.

$$X_{ES} = C X_S \quad (11)$$

$$\text{where } X_S = \begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix}$$

and  $x_S, y_S, z_S$  are the coordinates of the sun. Hence the coordinates of the sun in the OPEP system are:

$$X_{ES} = \begin{bmatrix} x_{ES} \\ y_{ES} \\ z_{ES} \end{bmatrix}$$

In terms of vector notation the sun vector in the OPEP system is:

$$\vec{S}_E = \vec{i}_E x_{ES} + \vec{j}_E y_{ES} + \vec{k}_E z_{ES} \quad (12)$$

The projection of the sun onto the OPEP  $x_E - y_E$  plane is simply:

$$\bar{S}_{E_P} = \bar{i}_E x_{E_S} + \bar{j}_E y_{E_S} \quad (13)$$

The angle  $C_E$  between the  $x_E$  axis and the projection of the sun on the OPEP  $x_E - y_E$  plane defined by:

$$C_E = \cos^{-1} \bar{i}_E \cdot \frac{\bar{S}_{E_P}}{|\bar{S}_{E_P}|} \quad (14)$$

### C. The Computer Program

The equations in Section II were used with an existing general purpose interplanetary trajectory Encke method program (ITEM) for use on the IBM 7090 and IBM 7094 computers (Ref. 6). The perturbations accounted for in this program include the gravitational attraction of the earth, moon, sun, Jupiter, Venus, and Mars considered as point masses. Also included are the effects of the 2nd, 3rd and 4th zonal harmonics and 2nd tesseral harmonic of the earth's gravitational field. Since perigee height rises rapidly, air drag was not considered. The perigee height rise for OGO-A is about forty-one kilometers per orbit.

### III. RESULTS AND DISCUSSION

Computer runs to determine  $\xi$ ,  $S'$ ,  $\delta$ ,  $\varphi_p$  and  $C_E$  as functions of time were made using the formulas of Section II and the following injection times:

August 7, 1966	1 <sup>h</sup> 45 <sup>m</sup>	universal time
August 7, 1966	3 <sup>h</sup> 45 <sup>m</sup>	universal time
August 22, 1966	1 <sup>h</sup> 15 <sup>m</sup>	universal time
August 22, 1966	3 <sup>h</sup> 15 <sup>m</sup>	universal time

#### A. Orbit Plane Geometry

##### 1. Inclination with respect to the Ecliptic Plane, $\xi$ .

Figures 4 and 5 show the trend of  $\xi$  for the above injection times. This angle exhibits the approximate fourteen day lunar oscillation and the bi-annual oscillation due to the solar perturbation. The mean increasing trend is due primarily to the combined long term effects of the sun and moon.

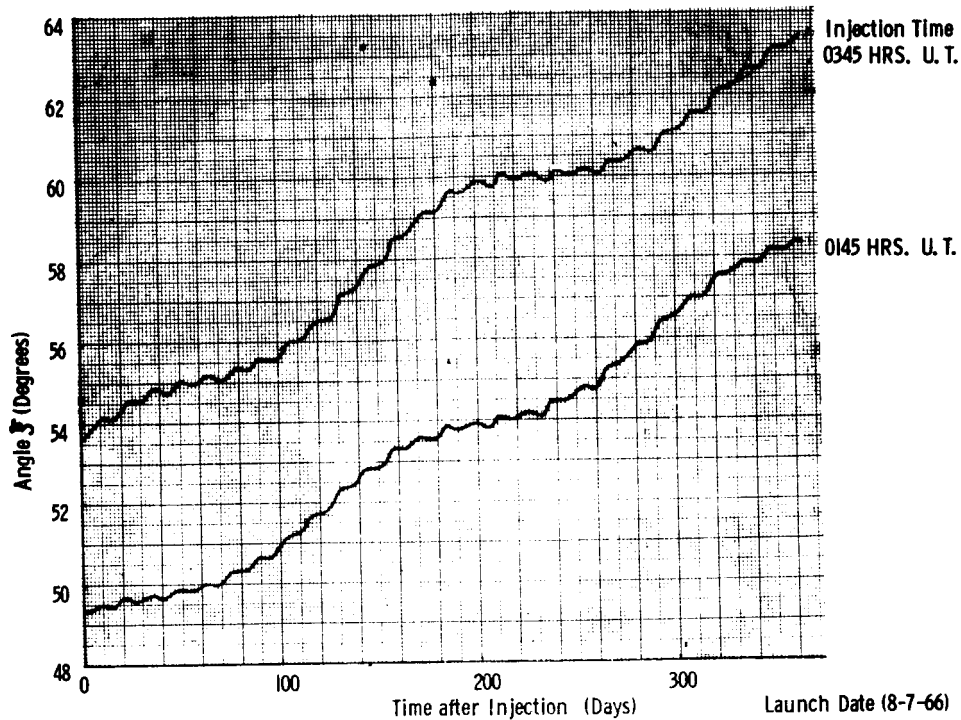


FIGURE 4 -  $\xi$  Versus Time after Injection

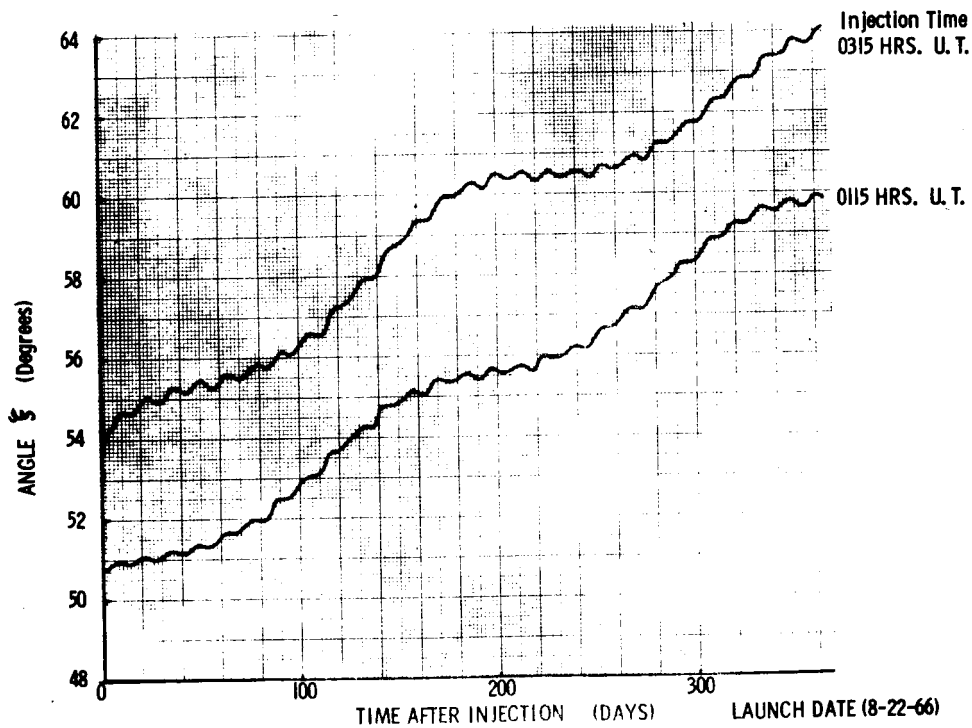


FIGURE 5 -  $\xi$  Versus Time after Injection

## 2. Angular Momentum - Sun Angle, $S'$

Since  $S'$  is mainly a function of the sun's position, the time history of  $S'$  shows a yearly periodicity. The short term periodic (about fourteen days) lunar perturbation also affects  $S'$  but these variations are in the order of tenths of a degree (about two orders of magnitude less than the average value of  $S'$ ) and, consequently, are not perceptible on Figures 6 and 7.



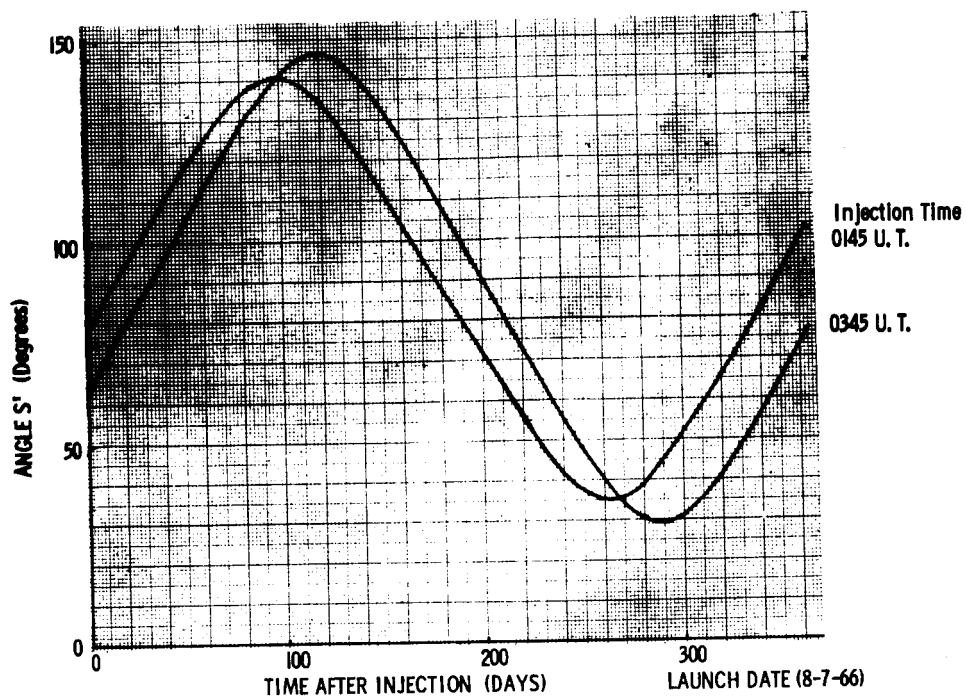


FIGURE 6- S' Versus Time After Injection

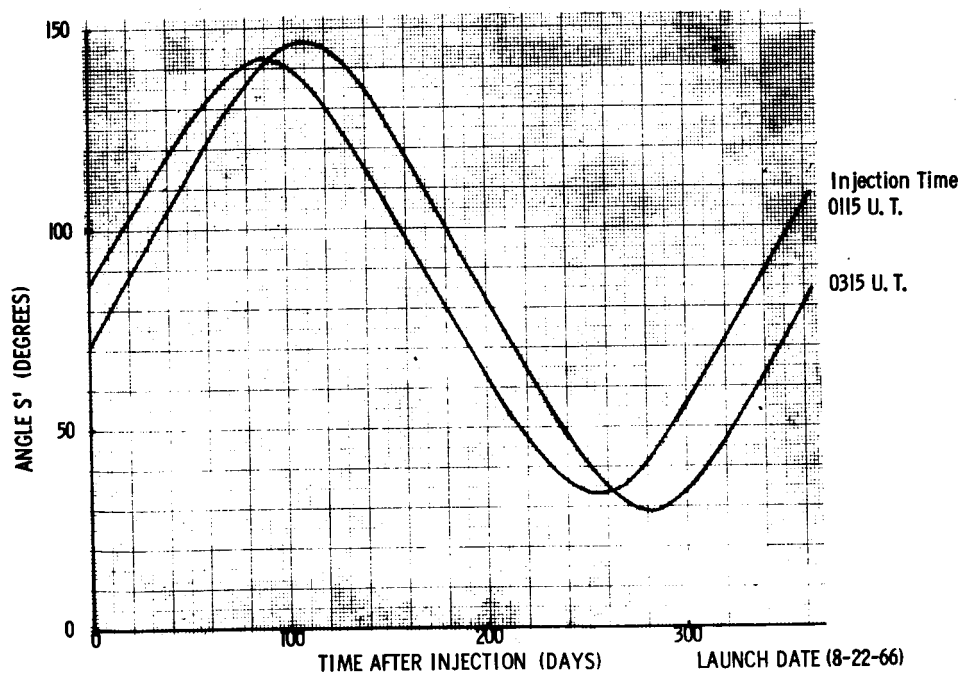


FIGURE 7 - S' Versus Time after Injection

## B. Spacecraft Angles

### 1. Half Angular Dimension of the Earth, $\delta$ .

The angle  $\delta$  is nearly periodic since it is a function solely of the distance of the spacecraft from the earth. The time history for the first 150 hours after injection is essentially the same for all the injection dates and times shown on page 15. See Figure 8.

After almost one year of flight the time history of  $\delta$  is again shown for 150 hours (Figure 9). These results are based on an injection into orbit time of August 7, 1966, at  $5^h 45^m$  universal time. The results shown on Figure 9 are typical of results for different injection dates and times.

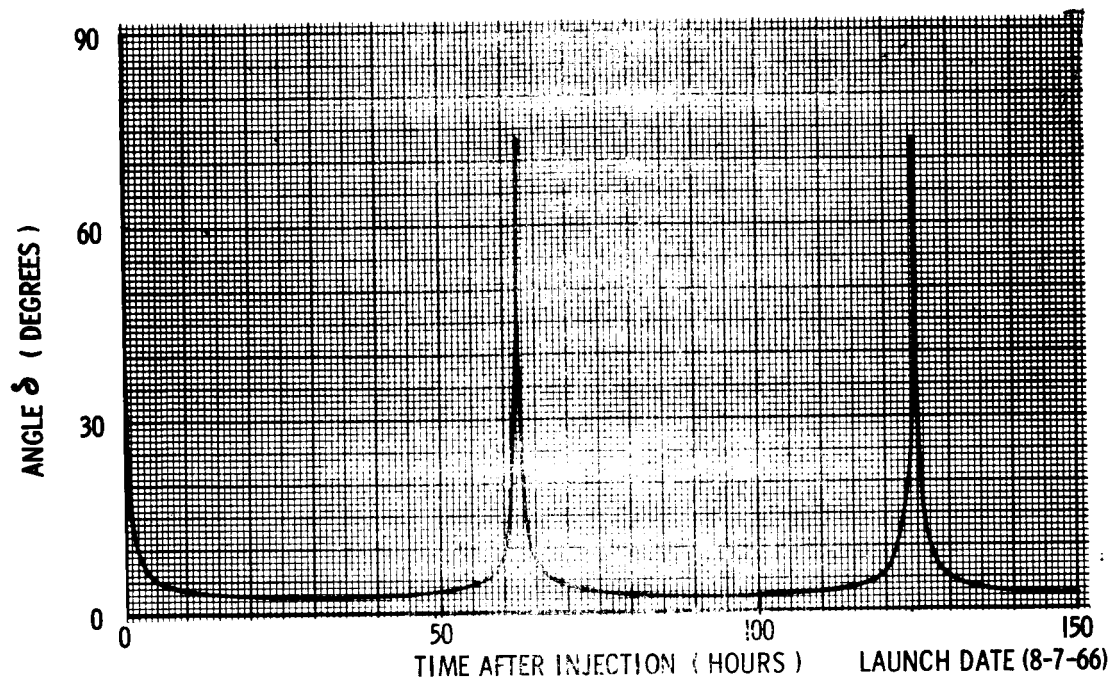


FIGURE 8 -  $\delta$  Versus Time after Injection

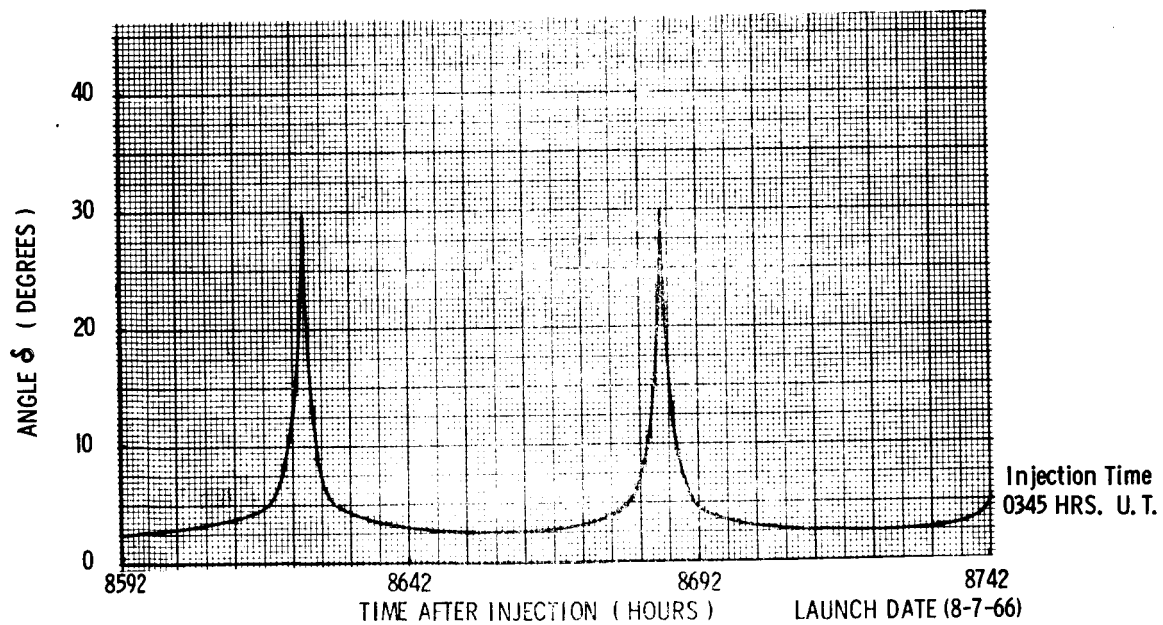


FIGURE 9 -  $\delta$  Versus Time after Injection

2. The Solar Array Angle,  $\phi_p$ .

Figures 10 and 11 show the behavior of the solar array angle  $\phi_p$  for the first 150 hours in orbit for the injection into orbit times shown on page 15. The different time histories are due to the different spacecraft-sun geometry.

Figures 12 and 13 show the behavior of the solar array angle  $\phi_p$  after almost a year in orbit.

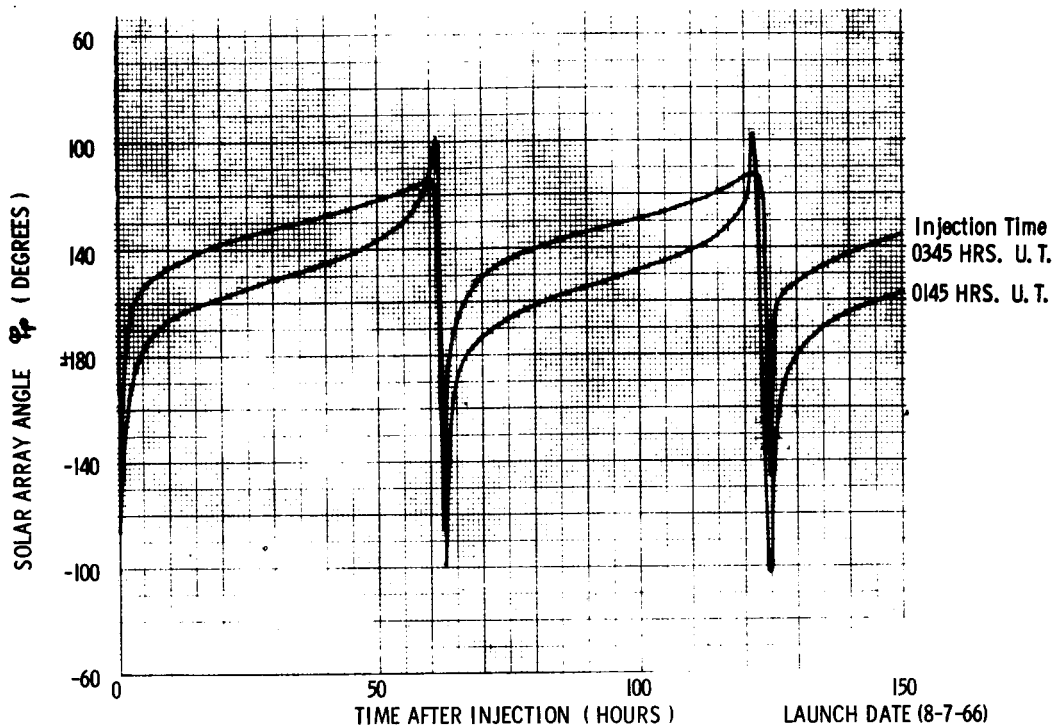


FIGURE 10 -  $\varphi_p$  Versus Time after Injection

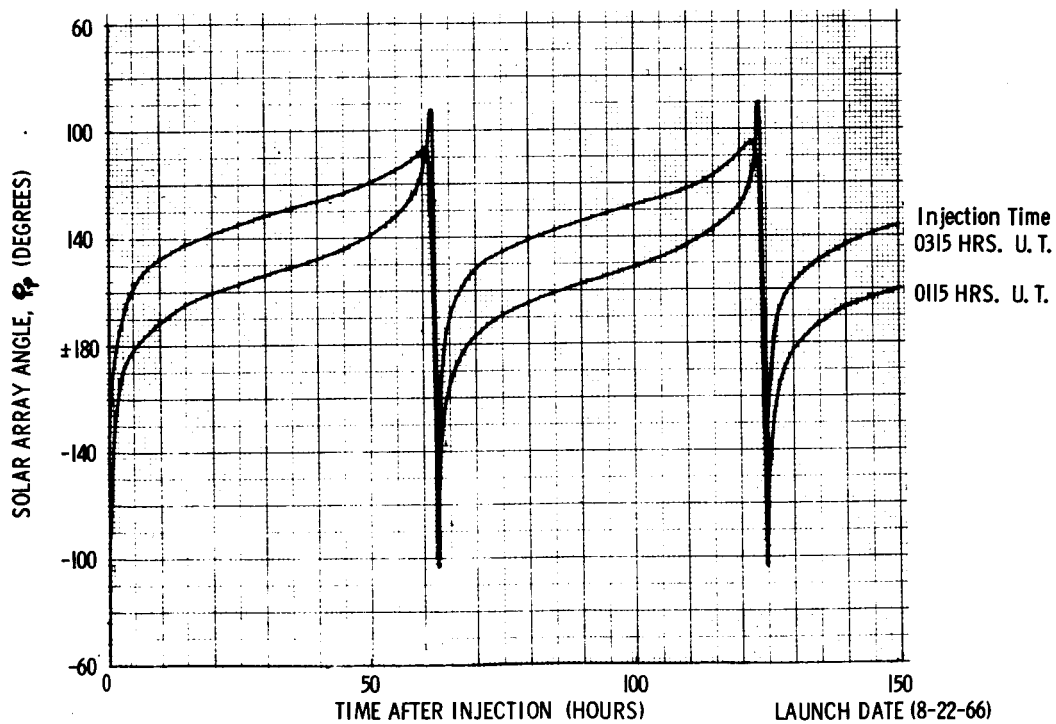


FIGURE 11 -  $\varphi_p$  Versus Time after Injection

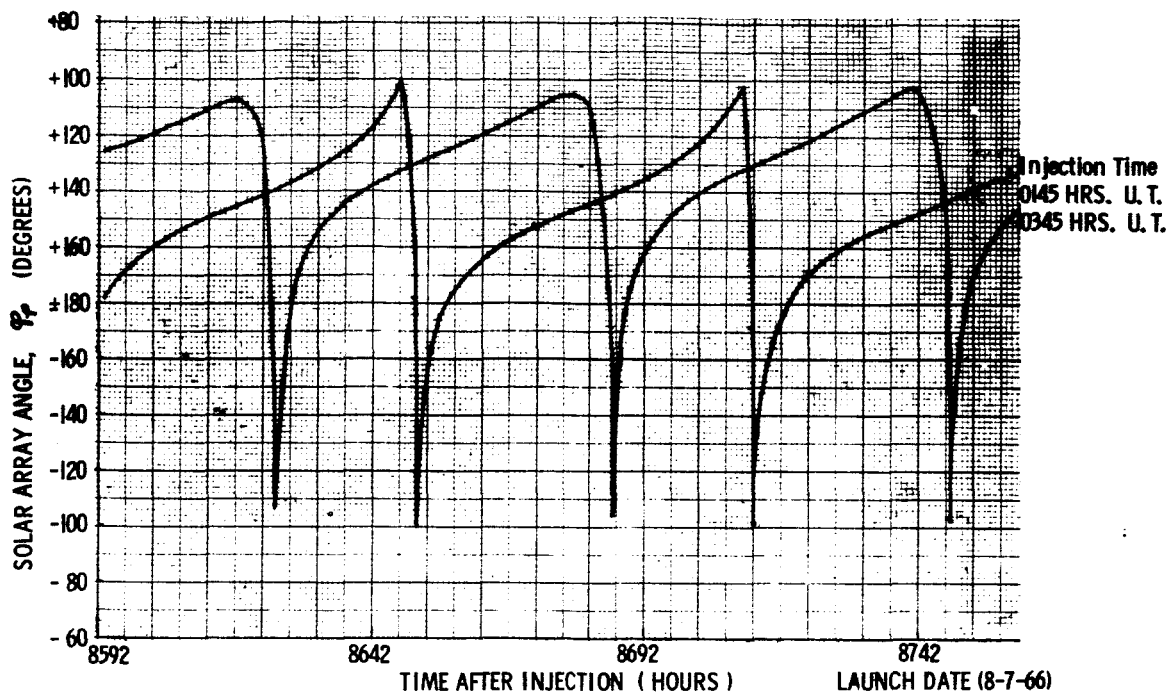


FIGURE 12 -  $\varphi_p$  Versus Time after Injection

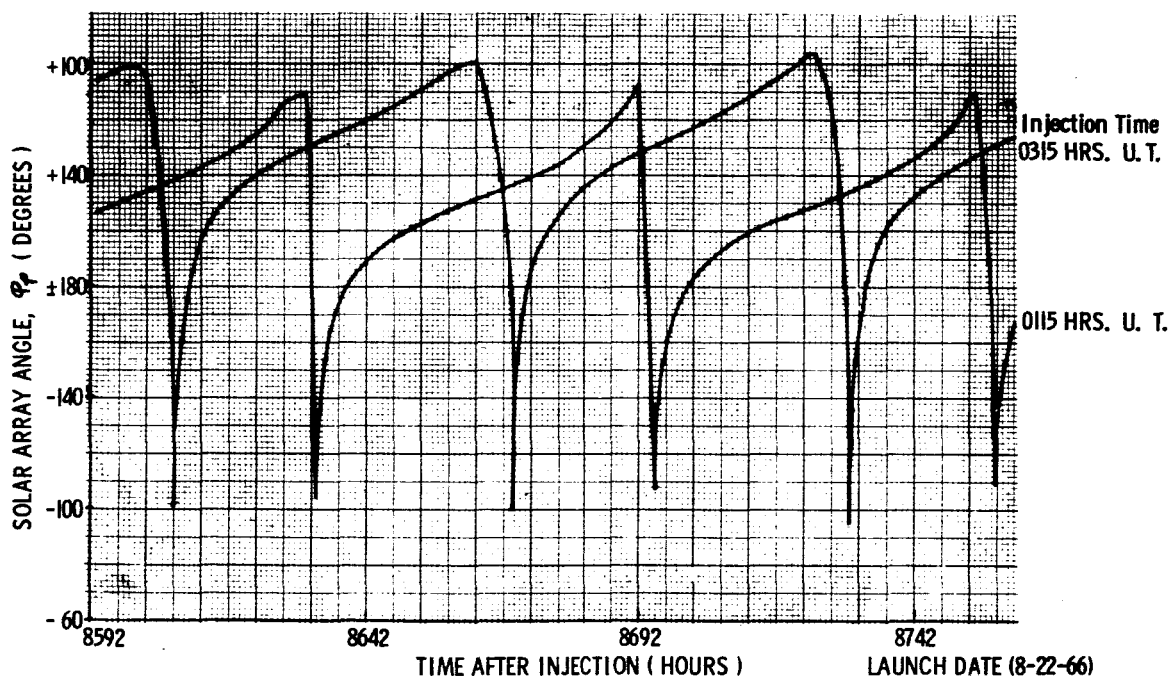


FIGURE 13 -  $\varphi_p$  Versus Time from Injection

3. OPEP  $x_E$ -Axis - Sun Projection Angle,  $C_E$ .

Figures 14 and 15 show the behavior of  $C_E$  for the first 150 hours in orbit for the injection into orbit times shown on page 15. As in the case of the solar array angle the difference in the time histories of  $C_E$  for different injection times is due to the different spacecraft-sun geometry.

Figures 16 and 17 show the behavior of  $C_E$  after almost a year in orbit.

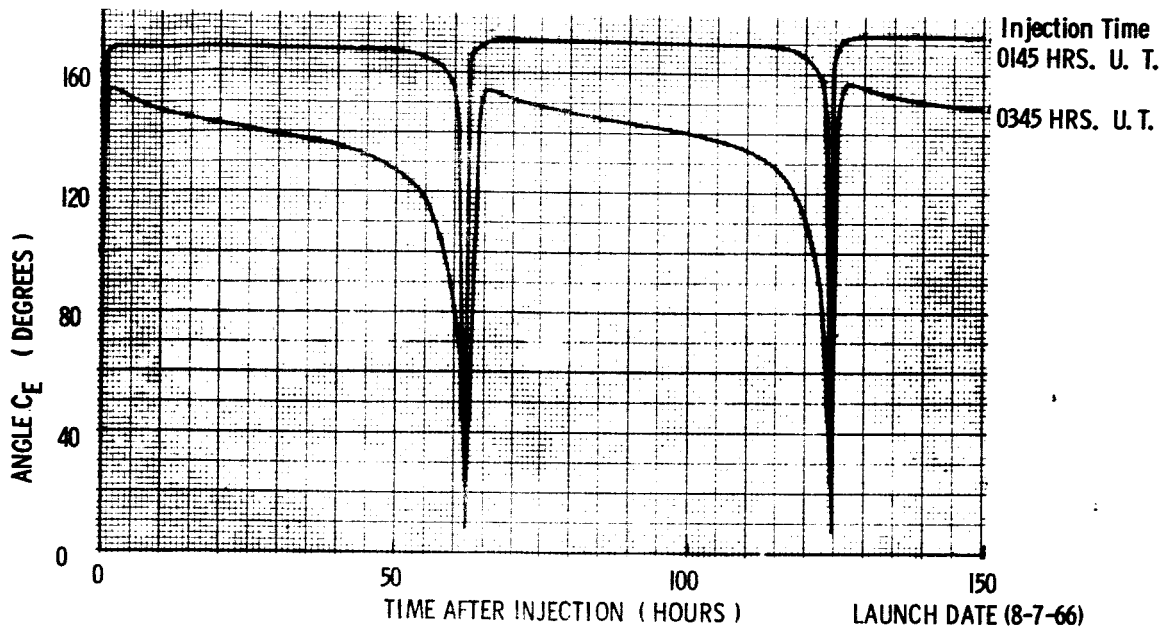


FIGURE 14 -  $C_E$  Versus Time after Injection

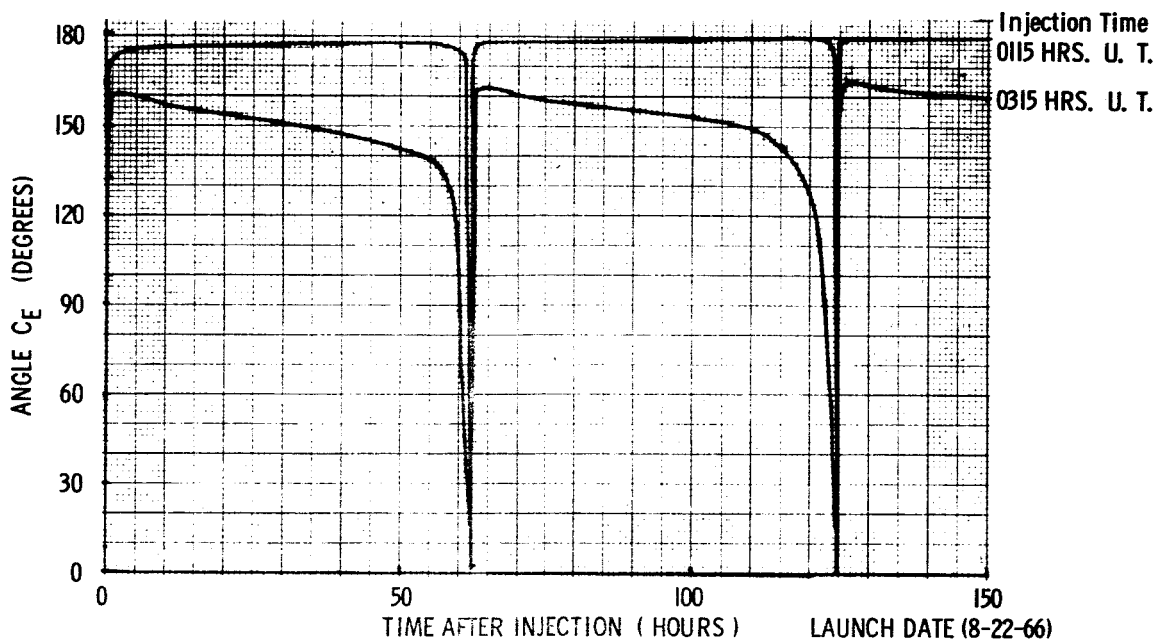


FIGURE 15 -  $C_E$  Versus Time after Injection



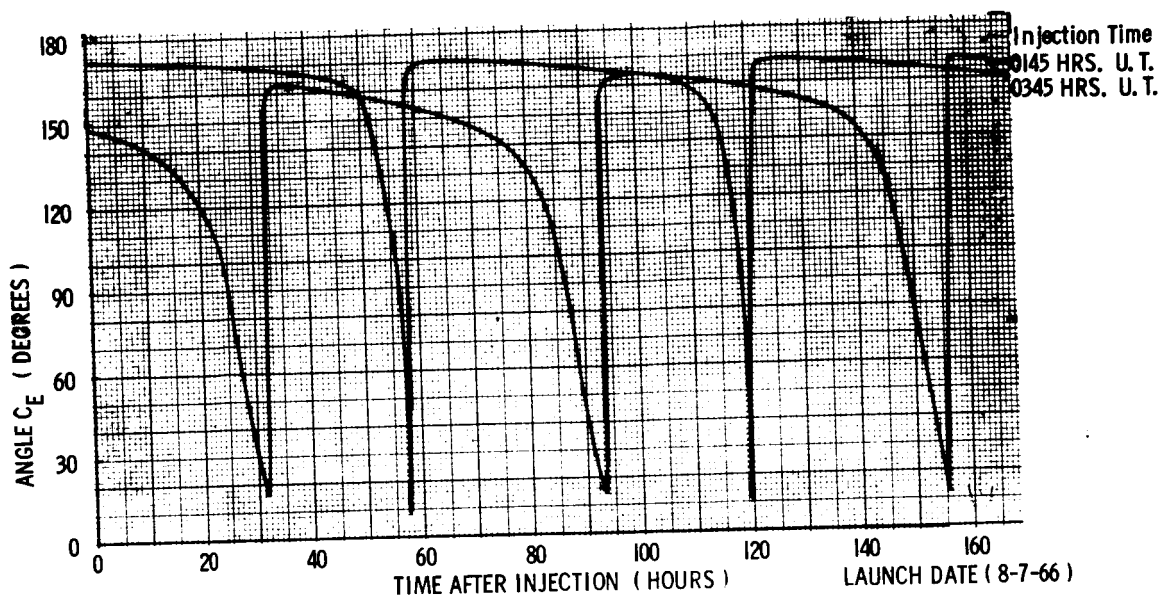


FIGURE 16 -  $C_E$  Versus Time after Injection

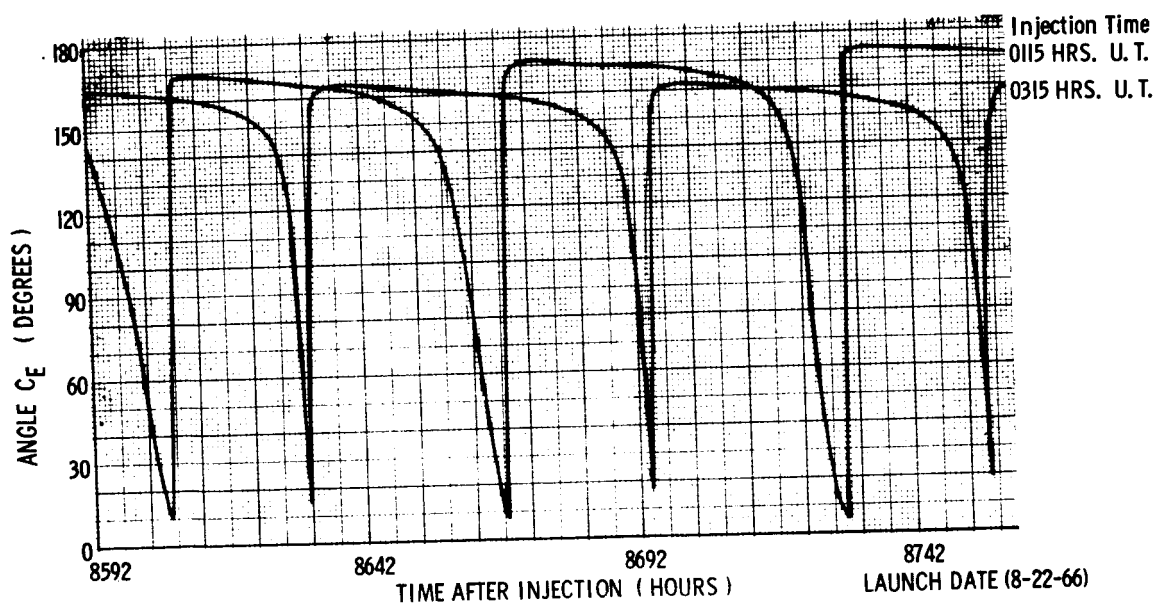


FIGURE 17 -  $C_E$  Versus Time after Injection

#### REFERENCES

1. TWX to R. E. Russey, Goddard Space Flight Center, from Lockheed Missile and Space Company, Sunnyvale, California, dated April 13, 1964. Subject: Injection Parameters for EOGO-1.
2. The American Ephemeris and Nautical Almanac for the Year 1966, (Washington: U. S. Government Printing Office, 1964).
3. Kaula, W. M., A Review of Geodetic Parameters, Document No. X-640-63-55 (Greenbelt, Maryland: Goddard Space Flight Center, 1963). This paper was presented at the International Astronomical Union Symposium No. 21 in Paris, France, May 27-31, 1963.
4. Otten, D. D., OGO Attitude Control Subsystem Description, Logic, and Specifications, Report No. 2313-0004-RV-000 (Redondo Beach, California: Space Technology Laboratories, Inc., December 1961).
5. Montgomery, H. E. and Paddack, S. J., EGO Orbital Parameters and Heat Inputs, Document No. X-643-63-61 (Greenbelt, Maryland: Goddard Space Flight Center, April 1963).
6. Shaffer, F., Squires, R. K., and Wolf, H., Interplanetary Trajectory Encke Method (ITEM) Program Manual, Document No. X-640-63-71 (Greenbelt, Maryland: Goddard Space Flight Center, May 1963).